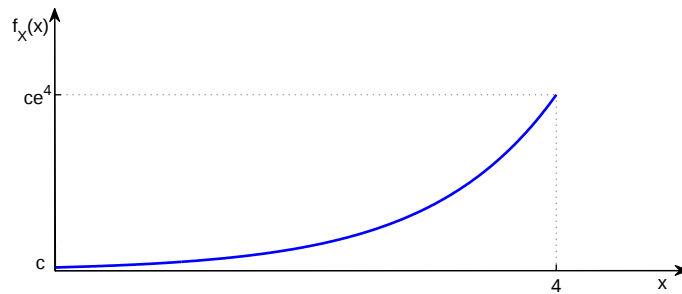


Θέμα 1ο

(α)



$$\text{Πρέπει: } \int_{-\infty}^{\infty} f_X(x) dx = 1 \Rightarrow \int_0^4 ce^x dx = 1 \Rightarrow ce^x \Big|_0^4 = 1 \Rightarrow c(e^4 - 1) = 1 \Rightarrow c = \frac{1}{e^4 - 1}$$

$$\begin{aligned} (\beta) \quad P(X \leq 3 | 2 \leq X \leq 6) &= \frac{P(X \leq 3 \cap 2 \leq X \leq 6)}{P(2 \leq X \leq 6)} = \frac{P(2 \leq X \leq 3)}{P(2 \leq X \leq 6)} = \frac{\int_2^3 ce^x dx}{\int_2^4 ce^x dx} = \\ &= \frac{e^3 - e^2}{e^4 - e^2} = \frac{e - 1}{e^2 - 1} = \frac{1}{e + 1} = 0.269 \end{aligned}$$

$$(\gamma) \quad P(2 \leq X \leq 6 | X > 3) = \frac{P(2 \leq X \leq 6 \cap X > 3)}{P(X > 3)} = \frac{P(3 < X \leq 6)}{P(3 < X \leq 6)} = 1$$

Θέμα 2ο

$$(\alpha) \quad P(F) = P(F|W)P(W) + P(F|W^c)P(W^c) = 0.1 \times 0.8 + 1 \times 0.2 = 0.28$$

$$(\beta) \quad P(W|F) = \frac{P(F|W)P(W)}{P(F)} = \frac{0.1 \times 0.8}{0.28} = \frac{8}{28} = \frac{2}{7} = 0.2857$$

$$(\gamma) \quad P(F_2|F_1) = \frac{P(F_2 F_1)}{P(F_1)}, \quad \text{όπου}$$

$$\begin{aligned} P(F_1 F_2) &= P(F_1 F_2 | W)P(W) + P(F_1 F_2 | W^c)P(W^c) \\ &= P(F_1 | W)P(F_2 | W)P(W) + P(F_1 | W^c)P(F_2 | W^c)P(W^c) \\ &\quad (\text{καθώς } F_1, F_2 \text{ υπό συνθήκη ανεξάρτητα δεδομένου του } W) \\ &= 0.1 \times 0.1 \times 0.8 + 1 \times 1 \times 0.2 = 0.208 \end{aligned}$$

Συνεπώς

$$P(F_2|F_1) = \frac{0.208}{0.28} = 0.743$$

και

$$P(W|F_1F_2) = \frac{P(F_1F_2|W)P(W)}{P(F_1F_2)} = \frac{0.008}{0.208} = 0.0385$$

Θέμα 3ο

$$(\alpha) \quad P(X \geq 15) = 1 - P(X \leq 15) = 1 - P\left(\frac{X - 10}{4} \leq \frac{15 - 10}{4}\right) = 1 - \Phi(5/4) = 0.1056$$

$$(\beta) \quad P(X \leq 5) = P\left(\frac{X - 10}{4} \leq \frac{5 - 10}{4}\right) = \Phi(-5/4) = 1 - \Phi(5/4) = 0.1056$$

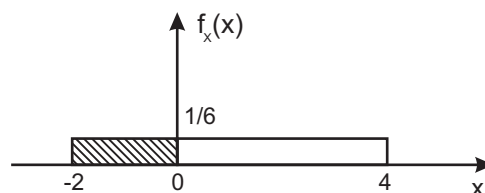
$$\begin{aligned} (\gamma) \quad P(X^2 \geq 400) &= P(X \geq 20 \cup X \leq -20) = P(X \geq 20) + P(X \leq -20) = \\ &= 1 - P\left(\frac{X - 10}{4} \leq \frac{20 - 10}{4}\right) + P\left(\frac{X - 10}{4} \leq \frac{-20 - 10}{4}\right) = 1 - \Phi(10/4) + \Phi(-30/4) = \\ &= 2 - \Phi(10/4) - \Phi(30/4) = 0.0062 \end{aligned}$$

$$(\delta) \quad P(X = 2) = 0 \text{ καθώς η τ.μ. } X \text{ είναι συνεχής.}$$

Θέμα 4ο

$$(\alpha) \quad \text{Καθώς } X \sim U[a, b], \quad E[X] = \frac{a+b}{2} = 1, \quad \text{Var}(X) = \frac{(b-a)^2}{12} = 3$$

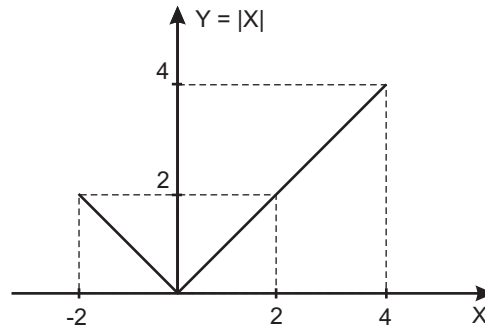
Λύνοντας, έχουμε ότι $a = -2$ και $b = 4$.



$$\text{Άρα, } P(X < 0) = \frac{1}{6} \times 2 = \frac{1}{3}$$

$$(\beta) \quad E[|X|] = \int_{-2}^0 -x \frac{1}{6} dx + \int_0^4 x \frac{1}{6} dx = -\frac{1}{6} \frac{x^2}{2} \Big|_{-2}^0 + \frac{1}{6} \frac{x^2}{2} \Big|_0^4 = \frac{5}{3}$$

(Υ)

(δ) Για $y \leq 0$, $F_Y(y) = 0$

$$\text{Για } 0 \leq y \leq 2, F_Y(y) = P(Y \leq y) = P(-y \leq X \leq y) = \int_{-y}^y \frac{1}{6} dx = \frac{y}{3}$$

$$\text{Για } y \geq 2, F_Y(y) = P(Y \leq y) = P(X \leq y) = \int_{-2}^y \frac{1}{6} dx = \frac{1}{6}(y+2)$$

$$\text{Συνεπώς, } F_Y(y) = \begin{cases} 0, & y \leq 0 \\ y/3, & 0 \leq y \leq 2 \\ (y+2)/6, & 2 \leq y \leq 4 \\ 1, & y \geq 4 \end{cases}$$

$$(\epsilon) \quad f_Y(y) = \frac{dF_Y(y)}{dy} = \begin{cases} 1/3, & 0 \leq y \leq 2 \\ 1/6, & 2 < y \leq 4 \\ 0, & \text{αλλού} \end{cases}$$